Hyperbolic Functions

Definitions

$\sinh(x) = \frac{e^x - e^{-x}}{2}$

$\cosh(x) = \frac{e^x + e^{-x}}{2}$

$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

$\operatorname{sech}(x) = \frac{2}{e^x + e^{-x}}$

$\operatorname{csch}(x) = \frac{2}{e^x - e^{-x}}$

Opposite Argument

$\sinh(-x) = -\sinh(x)$

$\cosh(-x) = \cosh(x)$

$\tanh(-x) = -\tanh(x)$

$\coth(-x) = -\coth(x)$

$\operatorname{sech}(-x) = \operatorname{sech}(x)$

$\operatorname{csch}(-x) = -\operatorname{csch}(x)$

Hyperbolic Identities

$\sinh(x) + \cosh(x) = e^x$

$\cosh^2(x) - \sinh^2(x) = 1$

$1 - \tanh^2(x) = \operatorname{sech}^2(x)$

$\coth^2(x) - 1 = \operatorname{csch}^2(x)$

Transformations

$\sinh(x) = \frac{2\tanh(\frac{x}{2})}{1 - \tanh^2(\frac{x}{2})}$

$\cosh(x) = \frac{1 + \tanh^2(\frac{x}{2})}{1 - \tanh^2(\frac{x}{2})}$

$\tanh(x) = \frac{2\tanh(\frac{x}{2})}{1 + \tanh^2(\frac{x}{2})}$

Double Argument Formulas

$\sinh(2x) = 2\sinh(x)\cosh(x)$

$\cosh(2x) = \cosh^2(x) + \sinh^2(x)$

$= 2\cosh^2(x) - 1$

$= 2\sinh^2(x) + 1$

$\tanh(2x) = \frac{2\tanh(x)}{1 + \tanh^2(x)}$

Triple Argument Formulas

$\sinh(3x) = 3\sinh(x) + 4\sinh^3(x)$

$\cosh(3x) = 4\cosh^3(x) - 3\cosh(x)$

$\tanh(3x) = \frac{\tanh^3(x) + 3\tanh(x)}{1 + 3\tanh^2(x)}$

Inverse Hyperbolic Funtions

$\operatorname{arcsinh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$

$\operatorname{arccosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$

$\operatorname{arctanh}(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

Half Argument Formulas

$\sinh(\frac{x}{2}) = \pm\sqrt{\frac{\cosh(x) - 1}{2}}$

$\cosh(\frac{x}{2}) = \pm\sqrt{\frac{\cosh(x) + 1}{2}}$

$\tanh(\frac{x}{2}) = \pm\sqrt{\frac{\cosh(x) - 1}{\cosh(x) + 1}}$

$= \frac{\cosh(x) - 1}{\sinh(x)}$

$= \frac{\sinh(x)}{1 + \cosh(x)}$

Linearization Formulas

$\sinh^2(x) = \frac{\cosh(2x) - 1}{2}$

$\cosh^2(x) = \frac{\cosh(2x) + 1}{2}$

$\tanh^2(x) = \frac{\cosh(2x) - 1}{\cosh(2x) + 1}$

Argument Addition Formulas

$\sinh(a+b) = \sinh(a)\cosh(b) + \sinh(b)\cosh(a)$

$\sinh(a-b) = \sinh(a)\cosh(b) - \sinh(b)\cosh(a)$

$\cosh(a+b) = \cos(a)\cosh(b) + \sinh(a)\sinh(b)$

$\cosh(a-b) = \cos(a)\cosh(b) - \sinh(a)\sinh(b)$

$\tanh(a+b) = \frac{\tanh(a) + \tanh(b)}{1 + \tanh(a)\tanh(b)}$

$\tanh(a-b) = \frac{\tanh(a) - \tanh(b)}{1 - \tanh(a)\tanh(b)}$

Product-To-Sum Formulas

$\sinh(a)\sinh(b) = \frac{1}{2}(\cosh(a+b) - \cosh(a-b))$

$\cosh(a)\cosh(b) = \frac{1}{2}(\cosh(a+b) + \cosh(a-b))$

$\sinh(a)\cosh(b) = \frac{1}{2}(\sinh(a+b) + \sinh(a-b))$

$\cosh(a)\sinh(b) = \frac{1}{2}(\sinh(a+b) - \sinh(a-b))$

Sum-To-Product Formulas

$\sinh(a) + \sinh(b) = 2\sinh(\frac{a+b}{2})\cosh(\frac{a-b}{2})$

$\sinh(a) - \sinh(b) = 2\sinh(\frac{a-b}{2})\cosh(\frac{a+b}{2})$

$\cosh(a) + \cosh(b) = 2\cosh(\frac{a+b}{2})\cosh(\frac{a-b}{2})$

$\cosh(a) - \cosh(b) = 2\sinh(\frac{a+b}{2})\sinh(\frac{a-b}{2})$